Recent Trends in Mathematical Sciences

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الجامعة الوطنية
National University of Science and Technology
للعلوم والتكنولوجيا

Outline

1. Introduction: The Importance and Evolving Role of Mathematics
   - Early Developments (Pre-17th Century)
   - 17th-19th Century: Calculus and More
   - 20th Century: The Rise of Abstract Mathematics and Computing
2. Key Trends and Recent Developments: Interdisciplinary and Applied Mathematics
   - Mathematical Biology and Medicine
   - Financial Mathematics and Economics
   - Machine Learning and Data Science
   - Quantum Computing and Information Theory
3. An Example: Mathematical Epidemiology
4. Future Directions
5. Concluding Remarks: The Role of Mathematics in Shaping the Future

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- Mathematics continues to evolve rapidly across a diverse range of fields, with significant advancements influencing both theoretical and applied domains.

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- Arithmetic and Geometry:
  - Ancient civilizations like the Egyptians, Babylonians, Greeks, and Indians developed mathematical methods for trade, construction, astronomy, and navigation.
  - Applications: Measuring land, building structures (e.g., pyramids), and tracking celestial bodies.
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An Example: Mathematical Epidemiology

- Mathematical epidemiology focuses on the spread and control of infectious diseases.

- It uses mathematical models to understand disease dynamics, predict outbreaks, and evaluate interventions

- I It could predict whether the disease will die out or persists in the population, via the calculation of

The basic reproduction number $\mathcal{R}_0$

The average/expected number of secondary infections caused by a single infectious individual during their entire infectious lifetime.

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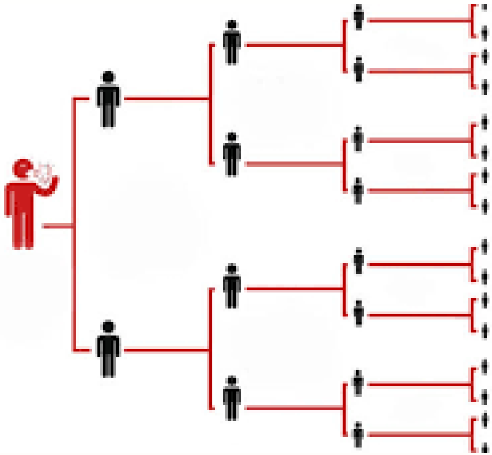
Illustration of $\mathcal{R}_0$



A Well-Known Result

- $\mathcal{R}0 < 1$: On average, infected individual produces less than one new infected individual over the course of its infectious period $\Longrightarrow$ infection will die out.
- $\mathcal{R}0 > 1$: each infected individual produces, on average more than one new infection $\Longrightarrow$ the disease can invade the population.
- $\mathcal{R}0 = 1$: sometimes it is a (forward or backward) bifurcation.

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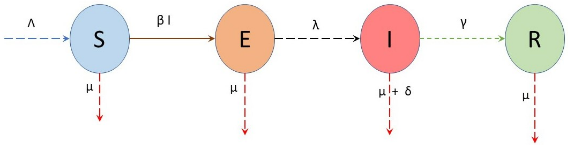
Illustrative Example: An SEIR Model



Consider the following SEIR model:

$$\begin{aligned} S'(t) &= \mu - \beta S I - \mu S \\ E'(t) &= \beta S I - (\alpha + \mu) I \\ I'(t) &= \alpha E - (\gamma + \mu) I \\ R'(t) &= \gamma I - \mu R \end{aligned}$$

for which

$$\mathcal{R}_0 = \frac{\alpha\beta}{(\alpha+\mu)(\gamma+\mu)}$$

Illustrative Example: An SEIR Model

If a portion ϵ of the new-born susceptible is vaccinated, then the model becomes:

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$$\hat{\mathcal{R}}_0 = \frac{(1-\epsilon)\alpha\beta}{(\alpha+\mu)(\gamma+\mu)} = (1-\epsilon)\mathcal{R}_0$$

Hence it is enough to vaccinate a percentage of $1 - \dfrac{1}{\mathcal{R}_0}$ to eliminate the disease.

Future Directions

- Advanced AI Models and Mathematical Foundations: More research is needed to understand the theory behind deep learning networks, including their capabilities and limitations.
- Advanced Mathematical Models for Climate Change: The urgent need to understand and mitigate climate change has driven interest in mathematical models of climate systems.
- Advanced Computational Techniques: With advances in hardware, we can now solve increasingly complex mathematical problems, pushing the boundaries of simulation and numerical methods.
- Unsolved Problems and Open Questions: like the Riemann Hypothesis, Navier-Stokes existence, and P vs NP.

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- Enhancing Interdisciplinary Collaboration: Mathematics is increasingly at the heart of interdisciplinary research. Collaboration between mathematicians and other scientists in various fields to solve real-world problems needs to be enhanced.
- Revisiting Mathematical Education: we should aim for an integrative approach that combines conceptual understanding, problem-solving, technological tools, and real-world applications along with fostering critical thinking, inclusivity, and effective communication.
- Final Thought: Mathematics continues to evolve and shape the world in unexpected ways—what seems abstract today may become the key to solving tomorrow's challenges.

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